requires minimal hardware and is readily implemented by modifying the gain in the rate gyro-loop with vehicle speed. For vehicles with low rudder-rates the relation between turning time and rudder-rate is displayed in Fig. 10. The finite slewing rate of the steering gear was shown to have little effect on maneuverability for normalized values of $\Omega_n = [(\Omega/A)/\eta](l/V)$ greater than 5. Design values of the required rudder-rate and controller gain ratio are given in Fig. 11 in terms of the vehicle constant $\eta = \Delta/JR/l$.

Finally, while this paper demonstrates that a precise model is not necessary to obtain meaningful results, such efforts should not supersede the continuing studies directed at refining dynamic models of floating and submerged vehicles.

In particular, this study does not consider the unique problems of vehicles with large rudders or highly maneuverable vehicles ($R/l = 1.5$) with special auxiliary equipment capable of producing additional turning forces, where precise knowledge of vehicle hydrodynamics is essential to the development of useful control systems.

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OCTOBER 1971 J. HYDRONAUTICS VOL. 5, NO. 4

Hydrodynamic Resistance of Towed Cables

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The hydrodynamic resistance on a towing cable is formulated on the basis of the independence principle which deduces that the chordwise viscous flow around a yawed cylinder is independent of the spanwise flow. The total resistance is resolved into the normal and the tangential resistances, respectively normal and tangential to the cable axis. The formulas of the normal and the tangential resistances can be universally applied to cables of any cross-sectional shape, and, for a given shape, require some empirical relationships that can be directly and conveniently obtained with a minimum of testing in a wind tunnel. This feature is attributable to the fact that the normal and the tangential resistances are expressed solely as functions of the Reynolds number based on the velocity component perpendicular to the axis of the cable. Working formulas for circular cables, derived from the universal formulas, yield values of the resistances that agree well with limited experimental data.

Nomenclature

C_D = drag coefficient, Eq. (4)
$[C_D]_f$ = frictional-drag coefficient
$[C_D]_p$ = pressure-drag coefficient
D = total drag per unit length of cable (or cylinder) oriented perpendicular to stream
d = diameter of circular or stranded cable in unstrained state
$\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ = unit vectors in the z_1-, z_2-, z_3-directions, respectively
$\mathbf{f}_f,\mathbf{f}_p$ = frictional and pressure resistances, respectively, per unit material length of cable
$\mathbf{f}_n$ = $f_n\mathbf{n}$, normal resistance per unit material length of cable
$\mathbf{f}_\tau$ = $f_\tau\boldsymbol{\tau}$, tangential resistance per unit material length of cable
h_1,h_2,h_3 = scale factors of z_1,z_2,z_3, respectively, Eq. (23)
h = local heat-transfer coefficient of fluid, Eq. (35)
$\bar{h}$ = average heat-transfer coefficient, Eq. (37)
k = thermal conductivity of fluid
L = characteristic length of cable
L_c = circumference of the cross section of cable, Eq. (38)
$\mathbf{n}$ = unit vector normal to the cable, Fig. 1
Nu = $\bar{h}L/k$, average Nusselt number
p = fluid pressure
Pr = $\mu c_p/k$, Prandtl number, where c_p is specific heat of fluid at constant pressure
$\mathbf{q}$ = heat-transfer rate per unit area
Re = $\rho LW/\mu$, Reynolds number
Re_n = $Re\cdot\sin\phi$
s = material coordinate, representing the arc length between the lower-end point and a material point P of cable in unstrained state
T = temperature in two-dimensional flow

Received September 21, 1970; revision received April 30, 1971.
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t = time
$\mathbf{U}$ = velocity of fluid at (s,t), due to ocean current
$\mathbf{V}$ = velocity of cable at (s,t)
$\mathbf{W}$ = fluid velocity relative to cable, Eq. (1)
W = $|\mathbf{W}|$, magnitude of $\mathbf{W}$
u,v,w = components in the x-, y-, z-directions, respectively, of fluid velocity in the neighborhood of cable
W_n = $W \sin\phi$
W_τ = $-W \cos\phi$
w_1,w_2,w_3 = components in the z_1-, z_2-, z_3-directions, respectively, of fluid velocity in the neighborhood of cable
x,y,z = Cartesian coordinates, Fig. 2
z_1,z_2,z_3 = orthogonal curvilinear coordinates, Fig. 2
ϵ = $\partial\sigma/\partial s - 1$, longitudinal strain in cable at (s,t)
μ = coefficient of viscosity of fluid
ρ = density of fluid
σ = actual length of cable between the lower-end point and a material point P in the general strained state at t
$\boldsymbol{\tau}$ = unit vector tangent to cable at (s,t), Fig. 1
τ_{23} = shear stress in z_2-direction, Eq. (21)
ϕ = angle between $\mathbf{W}$ and $\boldsymbol{\tau}$, Fig. 1

Subscripts

s = surface of cable
∞ = far away from cable, such that presence of cable is not felt

Introduction

In the past fifty years, many investigators developed models of the hydrodynamic resistance of towing cables. Casarella and Parsons[1] made an exhaustive survey of the past works. Some of the representative formulas currently used are developed by Landweber and Protter,[31] Pode,[2] Whicker,[3] and Eames.[4] They expressed the tangential or the normal resistance per unit length as a product of the drag of the cable per unit length when oriented normal to the fluid flow and a so-called loading function which is normally a polynomial of sine and cosine of the angle of the cable inclination to the flow. Springston[5] proposed a generalized form consisting of an infinite series of sine and cosine terms where the constants are determined by curve-fitting to specific test data. Wilson[6] expressed the component of the hydrodynamic resistance per unit length in one direction as a function of the velocity component in that direction and the drag coefficient depending on the Reynolds number based on it. Recently, using boundary-layer theory, Schram[7] and Calkins[8] formulated the resistances of faired cables. The objective of this report is to derive formulas of the normal and the tangential resistances, that require a minimum of wind-tunnel tests.

Preliminary Considerations

Let us assume that a material point P on the axis of a cable moves with a velocity $\mathbf{V}$ with respect to the axes fixed at some point on the ocean bottom (Fig. 1). The surrounding fluid may have a velocity $\mathbf{U}$ due to the ocean current. Then, the velocity of the fluid relative to the cable at the point P is given by

$$\mathbf{W} \equiv \mathbf{U} - \mathbf{V} \tag{1}$$

Let W denote magnitude of $\mathbf{W}$, that is,

$$W = |\mathbf{W}| \tag{2}$$

The important parameter of dynamic similarity is Reynolds number defined as

$$Re \equiv \rho L W/\mu \tag{3}$$

where ρ is the fluid density, μ the coefficient of viscosity of the fluid, and L the characteristic length of the cable. Usually, L is the diameter d of the cable. But L may be the chord length in the case of a faired cable as in aerodynamics. The values of temperature, density, and viscosity of sea-water range from -2–$38°$C, from nearly 1–1.07 gm/cm^{-3}, and from 0.007–0.019 gm/cm^{-1}/sec^{-1}, respectively. Taking the values of W between 1 and 25 knots, and those of d between $\frac{1}{16}$ and 2 in., and noting that thick cable usually operates at low speeds, we find that Reynolds number lies between 1.7×10^3 and 10^5 for circular and stranded-cables.

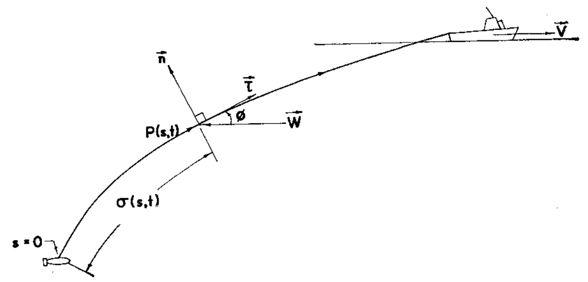

Fig. 1 Schematic diagram of a towed system.

See Fig. 1 which shows a schematic diagram of the towed system. Let s denote the arc length between the lower-end point and a material point P on the axis of the cable in some unstrained state. Note that s is not to be confused with the actual arc length σ of the cable between the lower-end point and the point P in its general strained state. To avoid confusion with σ, we refer to s as the material coordinate. Let ϵ denote the longitudinal strain in the cable at $P(s,t)$ where t is time. Then, $\epsilon = \partial\sigma/\partial s - 1$. When the cable is assumed to be inextensible, then $\sigma = s$ and $\epsilon = 0$.

In Fig. 1, $\boldsymbol{\tau}$ is defined as a unit vector tangent to the axis of the cable at $P(s,t)$ and in the direction of increasing s. Let ϕ denote the angle subtended by $\boldsymbol{\tau}$ and $\mathbf{W}$, measured counterclockwise from the $\mathbf{W}$-vector to the $\boldsymbol{\tau}$-vector. $\mathbf{n}$ is defined as a unit vector normal to the cable, that is obtained by rotating $\boldsymbol{\tau}$ counterclockwise through a right angle.

Inclination angle, strained cross-sectional shape, and relative velocity of fluid will generally vary along the axis of cable in actual operation. Only available information which can be applied here is Mrs. Surry's observation[9] that these effects of cable curvature are small when $0° \leq \phi \leq 60°$, as far as the normal resistance of circular cylinders is concerned. Therefore, in this report, the following assumption is made.

Basic Assumption

The hydrodynamic resistance acting on a cable element of arc length $\Delta\sigma$ is assumed to be the same as that acting on an element of length $\Delta\sigma$, of an infinitely long straight cable (also called cylinder hereafter) with its unstrained cross section and the same original diameter, inclined at the same angle to the stream of uniform relative fluid velocity.

In this report, the resistance per unit material length due to the shear stress at the cable surface is called the frictional resistance ($\mathbf{f}_f$) and the one due to the pressure is called the pressure resistance ($\mathbf{f}_p$). Conventionally, the total resistance per unit material length, $\mathbf{f}_f + \mathbf{f}_p$, is decomposed into the normal resistance ($\mathbf{f}_n$) and the tangential resistance ($\mathbf{f}_\tau$) as shown in Fig. 2. Hence, we have

$$\mathbf{f}_\tau + \mathbf{f}_n = \mathbf{f}_f + \mathbf{f}_p \tag{4}$$

Note that $\mathbf{f}_\tau$ is only a portion of $\mathbf{f}_f$, while $\mathbf{f}_n$ is the sum of $\mathbf{f}_p$ and the other portion of $\mathbf{f}_f$.

The frictional (or the pressure) resistance is termed the frictional (or the pressure) drag when the axis of the cable is placed normal to the stream of the fluid ($\phi = 90°$). Sum of the frictional and the pressure drag is called the total or the profile drag. The drag is customarily exprsssed in terms of the drag coefficient, defined as the dimensionless ratio

$$C_D = D/(\tfrac{1}{2}\rho W^2 L) \tag{5}$$

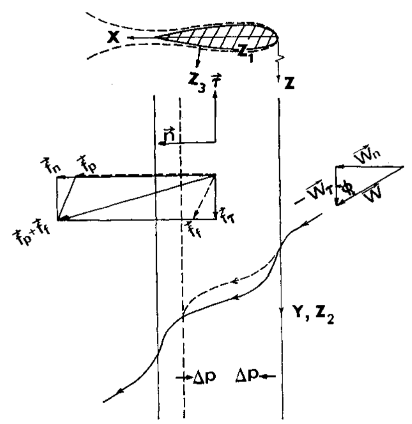

Fig. 2 Flow over a yawed infinite cylinder.

where D is the total drag per unit length of the cylinder. C_D's for geometrically similar bodies which have the same orientation with respect to the freestream direction, are functions of one variable only, namely the Reynolds number

$$C_D = C_D(Re) \tag{6}$$

Independence Principle

Let us consider an infinite cylinder at an angle ϕ to the relative flow, as shown in Fig. 2. Two coordinate systems are used—Cartesian coordinates x,y,z and orthogonal curvilinear coordinates z_1,z_2,z_3 (see Fig. 2). x is the distance measured from the leading edge along the chord, y is the distance measured parallel to the generators, and z is the distance measured from the xy-surface along a normal. Note that y and $\boldsymbol{\tau}$ are opposite to each other. z_1 is the distance measured along the surface in a direction perpendicular to the generators, z_2 is identical to y, and z_3 is the distance measured from the surface along a normal. Let u,v,w be the components in the x-, y-, z-directions, respectively, of fluid velocity of the viscous flow around the cable, and let w_1,w_2,w_3 be those in the z_1-, z_2-, z_3-directions, respectively.

Since the cylinder is infinite, any flow quantity is independent of y except for the case of vanishing chordwise flow. Therefore, the pressure gradient must be entirely chordwise as indicated in the figure. This implies that the pressure resistance acts in the cross-sectional plane.

With $\partial/\partial y \equiv 0$, the Navier-Stokes equations for incompressible flow are

$$\rho\left(u\,\frac{\partial u}{\partial x} + w\,\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{7}$$

$$\rho(u\partial v/\partial x + w\partial v/\partial z) = \mu(\partial^2 v/\partial x^2 + \partial^2 v/\partial z^2) \tag{8}$$

$$\rho\left(u\,\frac{\partial w}{\partial x} + w\,\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{9}$$

where p is pressure.

The continuity equation is

$$\partial u/\partial x + \partial w/\partial z = 0 \tag{10}$$

Eqs. (7, 9 and 10) constitute a problem in two-dimensional flow, which may be solved for u, w, and p without regard to v. Then, v is determined by the linear equation (8) with known variable coefficients u and w. This useful feature has been called the independence principle. Note that the independence principle means that the chordwise flow development is independent of the spanwise flow, not that the spanwise flow is independent of the chordwise flow.

The comparative simplicity of the analysis of laminar flow over yawed cylinders arises from the independence principle which is a direct consequence of the equations of motion. Both in theoretical and experimental work there is still some doubt whether turbulent flow, and the mechanisms producing transition to turbulence, are also independent of the spanwise flow. Bursnall and Loftin[10] made measurements of the pressure distribution about a yawed circular cylinder in the critical Reynolds number range, and found that the critical Reynolds number based on the velocity component perpendicular to the axis of the cylinder is roughly independent of ϕ from 90° down to 45°. A marked difference is shown in their results at $\phi = 30°$, however, and a reduction in the critical Reynolds number was observed. For turbulent flow, Young and Booth[11] and Altman and Hayter[12] say, on the basis of experiment, that the independence principle holds. Ashkenas and Riddell[13] on the other hand, have performed experiments on a yawed flat plate and concluded that the independence principle does not apply. Weber and Brebner,[14] assuming the independence principle for both laminar and turbulent boundary layers, estimated the profile drag of swept wings. Results show good agreement with experiment. From the above discussion we can conclude that the independence principle does not hold for small values of ϕ, for transition mechanisms, and for turbulent flow in the strict sense, but we may be able to obtain good engineering answers assuming that it does. In this report, we assume that the independence principle applies, that is, the flow in the cross-sectional plane is independent of the spanwise flow, regardless of whether the flow is laminar, turbulent or in transition.

Derivation of the Resistance Formulas

A logical way of deriving the expressions of the hydrodynamic resistances would be to solve Eqs. (7–10), obtain velocity components and pressure, formulate the pressure resistance ($\mathbf{f}_p$) and the frictional resistance ($\mathbf{f}_f$), and finally resolve $\mathbf{f}_p + \mathbf{f}_f$ into the normal and the tangential resistances. However, until the present day no general methods have become available for the integration of Eqs. (7–10). Asymptotic solutions for large and small Reynolds numbers respectively can offer only limited information even when these solutions can be obtained. Thus, to have complete, workable information, we must develop semiempirical relations which need some test results.

First, let us derive formulas of the normal resistance. Based on the independence principle, we only need to write an expression of f_n equivalent to Eqs. (5) and (6), that is,

$$f_n = C_D(Re_n)L\tfrac{1}{2}\rho W_n^2\cdot(1 + \epsilon) \quad \text{(any cable)} \tag{11}$$

where

$$Re_n \equiv \rho L W_n/\mu = Re\cdot\sin\phi \tag{12}$$

and

$$W_n = W\sin\phi \qquad W_\tau = -W\cos\phi \tag{13}$$

Note that the factor $(1 + \epsilon)$ in Eq. (11) is not necessary for inextensible cables. We can see from Eq. (11) that, for a given shape of the cable cross section, a table of C_D vs Re is all that is needed.

Working formula for circular cables is, by virtue of Eq. (11),

$$f_n = C_D(Re_n)d\tfrac{1}{2}\rho W_n^2(1 + \epsilon) \quad \text{(circular cables)} \tag{14}$$

with

$$C_D = [8\pi/(Re_n S)](1 - 0.87S^{-2}) \quad \text{(circular cables, } 0 < Re_n \leq 1\text{)} \tag{15}$$

where

$$S = -0.077215665 + \ln(8Re_n^{-1}) \tag{16}$$

and

$$C_D = 1.45 + 8.55\,Re_n^{-0.90} \quad \text{(circular cables, } 1 < Re_n < 30\text{)} \tag{17}$$

$$C_D = 1.1 + 4Re_n^{-1/2} \quad \text{(circular cables, } 30 \leq Re_n < 10^5\text{)} \tag{18}$$

Equation (15) is taken from the asymptotic solution by Kaplun.[15] It will be seen in Fig. 3 that the pressure-drag coefficient $[C_D]_p$ is nearly constant in the range $30 \leq Re \leq 10^5$ where it varies only between 0.9 and 1.1. On the other hand, the frictional-drag coefficient $[C_D]_f$ is found to be nearly equal to $4Re^{-1/2}$ in this range.[16] Then, for $30 \leq Re_n \leq 10^5$, we get Eq. (18) using

$$C_D \equiv [C_D]_p + [C_D]_f \tag{19}$$

Equation (17) for $1 < Re_n < 30$ is obtained by curve-fitting to experimental data.

Next, let us derive formula of the tangential resistance. There does not exist an expression similar to Eqs. (5) and (6). This expression will be sought in the following manner. The temperature $T(x,z)$ in low-speed, laminar, two-dimensional flow is governed by the equation

$$\rho(u\partial T/\partial x + w\partial T/\partial z) = \mu(\partial^2 T/\partial x^2 + \partial^2 T/\partial z^2) \tag{20}$$

when Prandtl number (Pr) of the fluid is unity.[17] Aerodynamicists have long been aware that Eq. (20) is identical with Eq. (8) for $v(x,z)$ of the viscous flow past a yawed cylinder. For example, Goland[29] and Frössling[30] used this similarity to determine the heat transfer across laminar boundary layers. We are going to use this analogy in the opposite way, that is, the skin friction in the spanwise direction of a yawed cylinder shall be related to the heat transfer at the surface of the same cylinder oriented normal to the flow.

The skin friction and the heat transfer at the surface $(z_3 = 0)$, denoted by the subscript s, can be expressed more simply in the orthogonal curvilinear coordinates z_1,z_2,z_3 than in the Cartesian coordinates x,y,z. Of course, Eqs. (8) and (20), written in the curvilinear coordinates, are identical to each other. Let us refer to these equations as Eqs. (8)′ and (20)′ without writing them out here because of their bulky nature.‡

Since w_2 is zero throughout the surface, total shear stress in the z_2-direction is given by

$$\tau_{23} = \mu \left\{ \frac{h_2}{h_3}\frac{\partial}{\partial z_3}\left(\frac{w_2}{h_2}\right) + \frac{h_3}{h_2}\frac{\partial}{\partial z_2}\left(\frac{w_3}{h_3}\right) \right\} \tag{21}$$

(p. 3 and p. 6 of Ref. 18). Here the scale factors h_i are defined as follows: if $\mathbf{r}$ represents the position vector of a point in space,

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{22}$$

where $\mathbf{i},\mathbf{j},\mathbf{k}$ are unit vectors in the x-, y-, z- directions, respectively, then the scale factor h_i is defined to be

$$h_i \equiv |\partial\mathbf{r}/\partial z_i| \qquad (i = 1,2,3) \tag{23}$$

Thus, the scale factors h_i can be determined from the relations between the two coordinate systems.[19] Since

$$\partial/\partial z_2 \equiv 0 \qquad h_2 \equiv 1 \tag{24}$$

Eq. (21) becomes

$$(\tau_{23})_s = \mu/h_3(\partial w_2/\partial z_3)_s \tag{25}$$

The heat-transfer rate $\mathbf{q}$ is, in general,

$$\mathbf{q} = -k\left(\frac{\mathbf{e}_1}{h_1}\frac{\partial T}{\partial z_1} + \frac{\mathbf{e}_2}{h_2}\frac{\partial T}{\partial z_2} + \frac{\mathbf{e}_3}{h_3}\frac{\partial T}{\partial z_3}\right) \tag{26}$$

where k is thermal conductivity of the fluid, and $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ are unit vectors in the z_1-, z_2-, z_3-directions, respectively. The first and the second term on the right-hand side of Eq. (26) vanish because the surface temperature T_s will be assumed to be constant throughout the surface. Thus, Eq. (26) is reduced to

$$\mathbf{q}_s = -k/h_3(\partial T/\partial z_3)_s\mathbf{e}_3 \tag{27}$$

Let us write

$$q_s = -k/h_3(\partial T/\partial z_3)_s \tag{28}$$

The boundary conditions for Eq. (8)′ are

$$\begin{aligned} v &= 0 \quad &\text{at} \quad z_3 &= 0 \\ v &\to -W_\tau \quad &\text{as} \quad z_3 &\to \infty \end{aligned} \tag{29}$$

and the boundary conditions for Eq. (20)′ are

$$\begin{aligned} T &= T_s \quad &\text{at} \quad z_3 &= 0 \\ T &\to T_\infty \quad &\text{as} \quad z_3 &\to \infty \end{aligned} \tag{30}$$

If v is replaced with $v/(-W_\tau)$ in Eq. (8)′ and if T with $(T - T_s)/(T_\infty - T_s)$ in Eq. (20)′, then

$$v/(-W_\tau) = (T - T_s)/(T_\infty - T_s) \tag{31}$$

and therefore both equations will yield the same numerical values.

Since f_τ is due to only skin friction as mentioned earlier, we have

$$-f_\tau = (1 + \epsilon) \oint_C (\tau_{23})_s dz_1 \tag{32}$$

Inserting Eq. (25) into Eq. (32), we get

$$-f_\tau = (1 + \epsilon) \oint_C (\mu/h_3)(\partial v/\partial z_3)_s dz_1 \tag{33}$$

where C denotes the circumference of the cable. Differentiating both sides of Eq. (31) with respect to z_3, and substituting the result into Eq. (33), we obtain

$$f_\tau = (1 + \epsilon)\mu W_\tau/(T_s - T_\infty) \oint_C -(1/h_3)(\partial T/\partial z_3)dz_1 \tag{34}$$

By definition, the local heat-transfer coefficient h is given by

$$h \equiv q_s/(T_s - T_\infty) = [-(k/h_3)(\partial T/\partial z_3)_s]/(T_s - T_\infty) \tag{35}$$

Substitution of Eq. (35) into Eq. (34) yields

$$f_\tau = (1 + \epsilon)\mu W_\tau \oint_C h/k \, dx \tag{36}$$

Defining the average heat-transfer coefficient $\bar{h}$ as

$$\bar{h} \equiv \oint_C (h/L_c)dx \tag{37}$$

where

$$L_c \equiv \oint_C dx = \text{circumference of the cable cross section} \tag{38}$$

and defining the average Nusselt number Nu as

$$Nu(Re_n) \equiv \bar{h}L/k \tag{39}$$

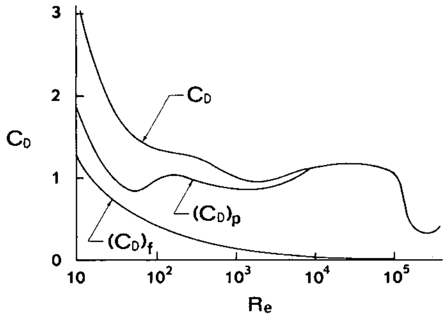

Fig. 3 Drag coefficient for circular cylinders.

‡ Interested readers may find full equations in p. 12 of Ref. 18 and reduce them using Eq. (24), or may transform Eqs. (8) and (20) with the help of the mathematical formulas of differential geometry.[19]

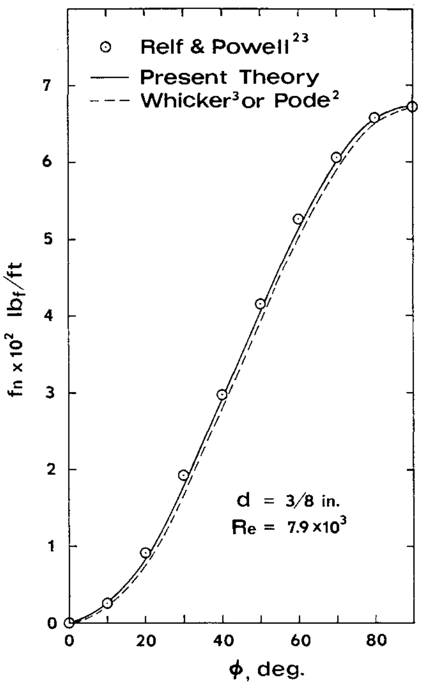

Fig. 4 Theoretical and experimental normal resistances of a circular cable.

we finally obtain

$$f_\tau = \frac{L_c}{L} \mu W_\tau Nu(Re_n) \cdot (1 + \epsilon) \quad \text{(any cable, } \phi \approx 0) \quad (40)$$

Again the factor $(1 + \epsilon)$ in Eq. (40) is not necessary for inextensible cables. Equation (40) means that the tangential component of the hydrodynamic resistance acting on an inclined cable can be obtained from the data of the heat transfer from the same straight cable, oriented perpendicular to the flow. Note that this is not a result of any hypothesis based on the diffusion of vorticity and heat, but a direct consequence of the equation governing the spanwise velocity in the laminar flow over a yawed cylinder and that governing the temperature in two-dimensional laminar flow past the same cylinder, placed perpendicular to the flow. Equation (11) for the normal resistance and Eq. (40) for the tangential resistance can be universally applied to cables of any shape.

Now, let us derive the working formula of the tangential resistance for circular cables from Eq. (40). There exist numerous experimental data of the convective heat transfer from circular cylinders. One of the best empirical relations between the average Nusselt number and the Reynolds number is due to Richardson,[20] and is given by

$$Nu(Re) = 0.55Re^{1/2} + 0.084Re^{2/3} \quad (41)$$

for Prandtl number equal to unity. Since $L_c = \pi d$ for circular cylinders, it follows from Eqs. (40) and (41) that

$$f_\tau = \pi\mu W_\tau(0.55Re_n^{1/2} + 0.084Re_n^{2/3}) \cdot (1 + \epsilon)$$
$$\text{(circular cables, } \phi \approx 0) \quad (42)$$

Discussion

The tangential resistance vanishes at $\phi = 0$ according to Eq. (40), whereas it has nonzero value at $\phi = 0$ in reality. Glauert and Lighthill[21] and Cebeci[22] demonstrate that the local tangential resistance per unit length of a circular cylinder, oriented parallel to the stream, depends on the distance from the nose as well as on Reynolds number. In this case, the flow is, at least, axisymmetric so that there is no force in cross-sectional planes. However, in the case of actual towing cable, the flow along the portion parallel to the relative fluid flow will not be axisymmetric so that a force may be brought about in the cross-sectional plane, because it will be affected by the nonaxisymmetric flow past the adjacent portions. Thus, the Basic Assumption itself is not satisfied in the first place. It was also stated earlier that the independence principle may not hold well for small values of ϕ. Here, it can only be said that the case of $\phi \approx 0$ must be analyzed with a different approach.

Figs. 4 and 5 contain a comparison between the resistance of circular cables given by Eqs. (14, 18, and 42) and those measured in a wind tunnel by Relf and Powell.[23] The agreement is seen to be good. For stranded cables, flow on the larger part of the surface may be turbulent because of roughness and grooves made by wires and strands. Therefore we may reason that, as compared with circular cables, $[C_D]_p$ will be smaller and $[C_D]_f$ will be larger for stranded cables. However, whether the total C_D is larger or smaller for stranded cables will not be known because, in available experimental data (Refs. 23, 25–28), C_D ranges from 0.7–2.2 in $3 \times 10^2 < Re < 4 \times 10^4$ and does not exhibit any trend. Thus, we may also have to use Eqs. (14–18 and 42) of circular cables for stranded cables until an exhaustive, systematic experiment is performed to discover the relation between the hydrodynamic resistance of stranded cables and Reynolds number $(0–10^5)$. No comparison can be made here for faired cables since existing experimental data are not available in open literature.

One noteworthy feature of the present theory is that the normal and the tangential components of the hydrodynamic resistance is explicitly expressed as functions of the Reynolds number based on the velocity component perpendicular to the axis of the cable. This fact indicates a unique way of obtaining the normal and the tangential resistances by experiment. Let us first review the restrictions imposed on the heat-transfer analogy, that is, the conditions under which Eq. (20) is valid. The condition of $Pr = 1$ excludes water channels and exclusively favors wind tunnels since $Pr = 0.71$ for air whereas Pr ranges from 2–13 for water below 212°F. The low speed means low Mach number such that the compressibility effects of air are negligible. Therefore, a typical wind-tunnel with a top speed of 300 ft/sec will serve well. Thus, by

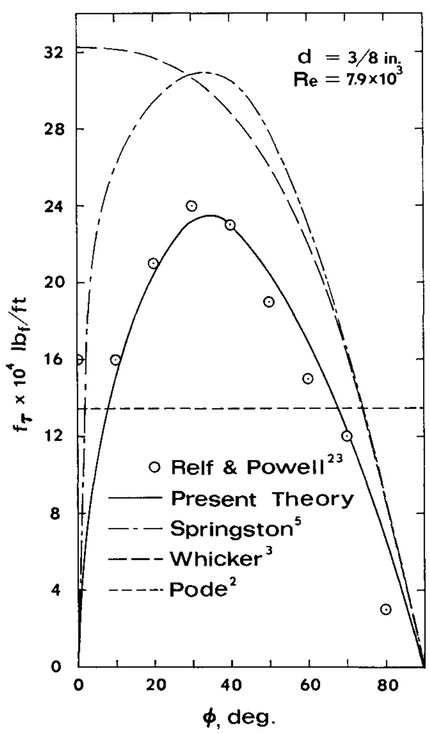

Fig. 5 Theoretical and experimental tangential resistances of a circular cable.

mounting a straight cable perpendicular to the stream in a wind-tunnel and obtaining a table of C_D and Nu vs Re, we can calculate the normal and the tangential resistances of the cable of any shape inclined to the flow using Eqs. (11) and (40).

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